Chapter 31: Nuclear Applications

Practice Problems

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1. The carbon isotope, $^{12}_{6}C$, has a nuclear mass of 12.0000 u.

   a. Calculate its mass defect.

6 protons = (6)(1.007825 u) =	6.046950	u
6 neutrons = (6)(1.008665 u) =	6.051990	u
total	12.098940	u
mass of carbon nucleus	− 12.000000	u
mass defect	−0.098940	u

   b. Calculate its binding energy in MeV.

   −(0.098940 u)(931.49 MeV/u)
   = −92.162 MeV

2. The isotope of hydrogen that contains 1 proton and 1 neutron is called deuterium. The mass of its nucleus is 2.0140 u.

   a. What is its mass defect?

1 proton =	1.007825	u
1 neutron =	1.008665	u
	2.016490	u
mass of deuterium nucleus =	− 2.0140	u
mass defect	−0.0025	u

   b. What is the binding energy of deuterium in MeV?

   −(0.0025 u)(931.49 MeV/u) = −2.3 MeV

3. A nitrogen isotope, $^{15}_{7}N$, has 7 protons and 8 neutrons. Its nucleus has a mass of 15.00011 u.

   a. Calculate the mass defect of this nucleus.

7 protons = 7(1.007825 u) =	7.054775	u
8 neutrons = 8(1.008665 u) =	8.069320	u
total	15.124095	u
mass of nitrogen nucleus	− 15.00011	u
mass defect	−0.123985	u
=	−0.12399	u

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   b. Calculate the binding energy of the nucleus.

   −(0.12399 u)(931.49 MeV/u) = −115.50 MeV

4. An oxygen isotope, $^{16}_{8}O$, has a nuclear mass of 15.99491 u.

   a. What is the mass defect of this isotope?

8 protons = (8)(1.007825 u) =	8.062600	u
8 neutrons = (8)(1.008665 u) =	8.069320	u
total	16.131920	u
mass of oxygen nucleus	− 15.99491	u
mass defect	−0.13701	u

   b. What is the binding energy of its nucleus?

   −(0.13701 u)(931.49 MeV/u) = −127.62 MeV

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5. Use Table C–5 of the Appendix to complete the following nuclear equations.

   a. $^{14}_{6}C \rightarrow ? + ^{0}_{-1}e$

   $^{14}_{6}C \rightarrow ^{14}_{7}N + ^{0}_{-1}e$

   b. $^{55}_{24}Cr \rightarrow ? + ^{0}_{-1}e$

   $^{55}_{24}Cr \rightarrow ^{55}_{25}Mn + ^{0}_{-1}e$

6. Write the nuclear equation for the transmutation of a uranium isotope, $^{238}_{92}U$, into a thorium isotope, $^{234}_{90}Th$, by emission of an alpha particle.

   $^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He$

7. A radioactive polonium isotope, $^{214}_{84}Po$, undergoes alpha decay and becomes lead. Write the nuclear equation.

   $^{214}_{84}Po \rightarrow ^{210}_{82}Pb + ^{4}_{2}He$

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8. Write the nuclear equations for the beta decay of these isotopes.

   a. $^{210}_{82}\text{Pb}$

   $^{210}_{82}\text{Pb} \rightarrow {}^{210}_{83}\text{Bi} + {}^{0}_{-1}e$

   b. $^{210}_{83}\text{Bi}$

   $^{210}_{83}\text{Bi} \rightarrow {}^{210}_{84}\text{Po} + {}^{0}_{-1}e$

   c. $^{234}_{90}\text{Th}$

   $^{234}_{90}\text{Th} \rightarrow {}^{234}_{91}\text{Pa} + {}^{0}_{-1}e$

   d. $^{239}_{93}\text{Np}$

   $^{239}_{93}\text{Np} \rightarrow {}^{239}_{94}\text{Pu} + {}^{0}_{-1}e$

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9. a. Calculate the mass defect for the deuterium–tritium fusion reaction used in the Tokamak, $^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + {}^{1}_{0}n$.

   Input masses 2.014102 u + 3.016049 u
   = 5.030151 u.
   Output masses 4.002603 u + 1.008665 u
   = 5.011268 u.
   Difference is –0.018883 u

   b. Find the energy equivalent of the mass defect.

   –(0.018883 u)(931.49 MeV/u)
   = –17.589 MeV

10. Calculate the energy released for the overall reaction in the sun where four protons produce one $^{4}_{2}\text{He}$, two positrons, and two neutrinos.

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Positron mass

$= (9.109 \times 10^{-31}\ \text{kg})\left[\frac{1\text{u}}{1.661\times10^{-27}\ \text{kg}}\right]$

= 0.000548 u

Input mass: 4 protons = 4(1.007825 u)
= 4.031300 u

Output mass:

$^{4}_{2}\text{He}$ + 2 positrons = 4.002603 u + 2(0.000548 u)
= 4.003699

Mass difference = 0.027601 u

Energy released = (0.027601 u)(931.49 MeV/u)
= 25.710 MeV

Chapter Review Problems

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1. A carbon isotope, $^{13}_{6}\text{C}$, has a nuclear mass of 13.00335 u.

   a. What is the mass defect of this isotope?

6 protons = (6)(1.007825 u) =	6.046950	u
7 neutrons = (7)(1.008665 u) =	7.060655	u
total	13.107605	u
mass of carbon nucleus	– 13.00335	u
mass defect	–0.10426	u

   b. What is the binding energy of its nucleus?

   –(0.10426 u)(931 MeV/u) = –97.1 MeV

2. A nitrogen isotope, $^{14}_{7}\text{N}$, has a nuclear mass of approximately 14.00307 u.

   a. What is the mass defect of the nucleus?

7 protons = (7)(1.007825 u) =	7.054775	u
7 neutrons = (7)(1.008665 u) =	7.060655	u
total	14.11543	u
mass of nitrogen nucleus	– 14.00307	u
mass defect	–0.11236	u

b. What is the binding energy of this nucleus?

$-(0.11236\ \text{u})(931\ \text{MeV/u}) = -105\ \text{MeV}$

c. What is the binding energy per nucleon?

$$Energy = \frac{-105\ \text{MeV}}{14\ \text{nucleons}}$$
$$= -7.5\ \text{MeV/nucleon}$$

3. A nitrogen isotope, ${}^{12}_{7}\text{N}$, has a nuclear mass of 12.0188 u.

a. What is the binding energy per nucleon?

7 protons = (7)(1.007825 u) =	7.054775	u
5 neutrons = (5)(1.008665 u) =	5.043325	u
total	12.098100	u
mass of nitrogen–12 nucleus	− 12.0188	u
mass defect	−0.0793	u

binding energy
$-(0.0793\ \text{u})(931\ \text{MeV/u}) = -73.8\ \text{MeV}$

$$\text{binding energy per nucleon} = \frac{-73.8\ \text{MeV}}{12\ \text{nucleons}}$$
$$= -6.15\ \text{MeV}$$

b. Does it require more energy to separate a nucleon from a ${}^{14}_{7}\text{N}$ nucleus or from a ${}^{12}_{7}\text{N}$ nucleus? Refer to the previous problem.

It requires more energy to remove a nucleon from nitrogen – 14, 7.5 MeV.

4. The two positively–charged protons in a helium nucleus are separated by about 2.0×10^{-15} m. Use Coulomb's law to find the electric force of repulsion between the two protons. The result will give you an indication of the strength of the strong nuclear force.

$$F = K\frac{qq'}{d^2}$$
$$= \frac{K(1.6 \times 10^{-19}\ \text{C})(1.6 \times 10^{-19}\ \text{C})}{(2.0 \times 10^{-15}\ \text{m})^2}$$
$$= 58\ \text{N}$$

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5. A ${}^{232}_{92}\text{U}$ nucleus, mass = 232.0372 u, decays to ${}^{228}_{90}\text{Th}$, mass = 228.0287 u, by emitting an α particle, mass = 4.0026 u, with a kinetic energy of 5.3 MeV. What must be the kinetic energy of the recoiling thorium nucleus?

mass defect
= (232.0372 u) – (228.0287 u + 4.0026 u)
= 0.0059 u

total KE = (0.0059 u)(931 MeV/u)
= 5.5 MeV

KE for thorium nucleus
= (5.5 MeV) – (5.3 MeV)
= 0.2 MeV

6. The binding energy for ${}^{4}_{2}\text{He}$ is 28.3 MeV. Calculate the mass of a helium nucleus in atomic mass units.

$$\text{Mass defect } \frac{28.3\ \text{MeV}}{931\ \text{MeV/u}} = 0.0304\ \text{u}$$

2 protons = (2)(1.007825 u) =	2.015650	u
2 neutrons = (2)(1.008665 u) =	2.017330	u
mass	4.032980	u
minus mass defect	− 0.0304	u
	4.0026	u

7. The radioactive nucleus indicated in each equation disintegrates by emitting a positron. Complete each nuclear equation.

a. ${}^{21}_{11}\text{Na} \rightarrow\ ?\ + {}^{0}_{+1}e + ?$

${}^{21}_{11}\text{Na} \rightarrow {}^{21}_{10}\text{Ne} + {}^{0}_{+1}e + {}^{0}_{0}\nu$

b. ${}^{49}_{24}\text{Cr} \rightarrow\ ?\ + {}^{0}_{+1}e + ?$

${}^{49}_{24}\text{Cr} \rightarrow {}^{49}_{23}\text{V} + {}^{0}_{+1}e + {}^{0}_{0}\nu$

8. Complete the nuclear equations for these transmutations.

a. ${}^{30}_{15}\text{P} \rightarrow\ ?\ + {}^{0}_{+1}e + ?$

${}^{30}_{15}\text{P} \rightarrow {}^{30}_{14}\text{Si} + {}^{0}_{+1}e + {}^{0}_{0}\nu$

b. $^{205}_{82}Pb \rightarrow\ ?\ + {}^{0}_{+1}e + ?$

$^{205}_{82}Pb \rightarrow {}^{205}_{81}Tl + {}^{0}_{+1}e + {}^{0}_{0}\upsilon$

9. A mercury isotope, $^{200}_{80}Hg$, is bombarded the deuterons, $^{2}_{1}H$. The mercury nucleus absorbs the dueteron and then emits an alpha particle.

a. What element is formed by this reaction?

Gold.

b. Write the nuclear equation for the reaction.

$^{200}_{80}Hg + {}^{2}_{1}H \rightarrow {}^{198}_{79}Au + {}^{4}_{2}He$

10. When bombarded by protons, a lithium isotope, $^{7}_{3}Li$, absorbs a proton and then ejects two alpha particles. Write the nuclear equation for this reaction.

$^{7}_{3}Li + {}^{1}_{1}H \rightarrow 2{}^{4}_{2}He$

11. Each of the nuclei given below can abosorb an alpha particle. Assume that no secondary particles are emitted by the nucleus. Complete each equation.

a. $^{14}_{7}N + {}^{4}_{2}He \rightarrow\ ?$

$^{14}_{7}N + {}^{4}_{2}He \rightarrow {}^{18}_{9}F$

b. $^{27}_{13}Al + {}^{4}_{2}He \rightarrow\ ?$

$^{27}_{13}Al + {}^{4}_{2}He \rightarrow {}^{31}_{15}P$

12. In each of these reactions, a neutron is absorbed by a nucleus. The nucleus then emits a proton. Complete the equations.

a. $^{65}_{29}Cu + {}^{1}_{0}n \rightarrow\ ?\ + {}^{1}_{1}H$

$^{65}_{29}Cu + {}^{1}_{0}n \rightarrow {}^{65}_{28}Ni + {}^{1}_{1}H$

b. $^{14}_{7}N + {}^{1}_{0}n \rightarrow\ ?\ + {}^{1}_{1}H$

$^{14}_{7}N + {}^{1}_{0}n \rightarrow {}^{14}_{6}C + {}^{1}_{1}H$

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13. When a boron isotope, $^{10}_{5}B$, is bombarded with neutrons, it absorbs a neutron and then emits an alpha particle.

a. What element is also formed?

Lithium.

b. Write the nuclear equation for this reaction.

$^{10}_{5}B + {}^{1}_{0}n \rightarrow {}^{7}_{3}Li + {}^{4}_{2}He$

14. When a boron isotope, $^{11}_{5}B$, is bombarded with protons, it absorbs a proton and emits a neutron.

a. What element is formed?

Carbon.

b. Write the nuclear equation for this reaction.

$^{11}_{5}B + {}^{1}_{1}p \rightarrow {}^{11}_{6}C + {}^{1}_{0}n$

c. The isotope formed is radioactive, decaying by emitting a positron. Write the complete nuclear equation for this reaction.

$^{11}_{6}C \rightarrow {}^{11}_{5}B + {}^{0}_{1}e + {}^{0}_{0}\upsilon$

15. The isotope most commonly used in *PET* scanners is $^{18}_{9}F$.

a. What element is formed by the positron emission of this element?

Oxygen.

b. Write the equation for this reaction.

$^{18}_{9}F \rightarrow {}^{18}_{8}O + {}^{0}_{1}e + {}^{0}_{0}\upsilon$

c. The half-life of $^{18}_{9}F$ is 110 min. A solution containing 10.0 mg of this isotope is injected into a patient at 8:00 a.m. How much remains at 3:30 p.m.?

The time is about 4 half-lives, so

$\frac{1}{16}(10.0\ \text{mg})$ or 0.63 mg remains.

Chapter Review Problems

16. The basements of many homes contain the rock granite or are built over granite. Granite contains small amounts of radioactive uranium and thorium. Uranium–238 goes through a series of decays before reaching a stable lead isotope. $^{238}_{92}U$ emits an alpha, leaving an isotope that decays by emitting a beta. The next decay is again a β, followed by three α decays. The result is the gas radon that can seep into the basement. Write the equations for these decays.

$^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He$

$^{234}_{90}Th \rightarrow ^{234}_{91}Pa + ^{0}_{-1}e + ^{0}_{0}\bar{\upsilon}$

$^{234}_{91}Pa \rightarrow ^{234}_{92}U + ^{0}_{-1}e + ^{0}_{0}\bar{\upsilon}$

$^{234}_{92}U \rightarrow ^{230}_{90}Th + ^{4}_{2}He$

$^{230}_{90}Th \rightarrow ^{226}_{88}Ra + ^{4}_{2}He$

$^{226}_{88}Ra \rightarrow ^{222}_{86}Rn + ^{4}_{2}He$

17. The first atomic bomb released an energy equivalent of 20 kilotons of TNT. One kiloton of TNT is the equivalent of 5.0×10^{12} J. What was the mass of the uranium-235 that was fissioned to produce this energy?

$Energy = (20 \text{ kilotons})(5.0 \times 10^{12} \text{ J/kiloton}) = 1.0 \times 10^{14} \text{ J}$

$$\frac{1.0 \times 10^{14} \text{ J}}{1.6 \times 10^{-19} \text{ J/eV}} = 6.3 \times 10^{32} \text{ eV}$$

$$\frac{(6.3 \times 10^{32} \text{ eV})}{(2.0 \times 10^{2} \text{ MeV/atom})(10^{6} \text{ eV/MeV})}$$

$= 3.1 \times 10^{24}$ atoms

$$\frac{3.1 \times 10^{24} \text{ atoms}}{6.02 \times 10^{23} \text{ atoms/mole}} = 5.2 \text{ moles}$$

$(5.2 \text{ moles})(0.235 \text{ kg/mole}) = 1.2 \text{ kg}$

18. Complete the following fission reaction.
$^{239}_{94}Pu + ^{1}_{0}n \rightarrow ^{137}_{52}Te + \ ? \ + 3^{1}_{0}n$

$^{239}_{94}Pu + ^{1}_{0}n \rightarrow ^{137}_{52}Te + ^{100}_{42}Mo + 3^{1}_{0}n$

19. Complete the following fission reaction.
$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{92}_{36}Kr + \ ? \ + 3^{1}_{0}n$

$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{92}_{36}Kr + ^{141}_{56}Ba + 3^{1}_{0}n$

20. Plutonium is formed in two steps. First U–238 absorbs a neutron which then becomes unstable U–239. The U–239 emits a beta particle forming a new element and this new element emits a second beta particle forming plutonium. Complete the following reaction.

$^{238}_{92}U + ^{1}_{0}n \rightarrow \left[^{239}_{92}U\right] \rightarrow$

$^{238}_{92}U + ^{1}_{0}n \rightarrow \left[^{239}_{92}U\right] \rightarrow ^{239}_{93}Np + ^{0}_{-1}e + ^{0}_{0}\bar{\upsilon}$

$^{239}_{93}Np \rightarrow ^{239}_{94}Pu + ^{0}_{-1}e + ^{0}_{0}\bar{\upsilon}$

21. Complete each of the following fusion reactions.

a. $^{2}_{1}H + ^{2}_{1}H \rightarrow \ ? \ + ^{1}_{0}n$

b. $^{2}_{1}H + ^{2}_{1}H \rightarrow \ ? \ + ^{1}_{1}H$

c. $^{2}_{1}H + ^{3}_{1}H \rightarrow \ ? \ + ^{1}_{0}n$

$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + ^{1}_{0}n$

$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{1}H + ^{1}_{1}H$

$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n$

22. One fusion reaction is $^{2}_{1}H + ^{2}_{1}H \rightarrow ^{4}_{2}He$.

a. What energy is released in this reaction?

$(2)(2.01410 \text{ u}) - (4.00263 \text{ u}) = 0.02557 \text{ u}$
$(0.02557 \text{ u})(931 \text{ MeV/u}) = 23.8 \text{ MeV}$

b. Deuterium exists as a diatomic, two–atom, molecule. One mole of deuterium contains 6.02×10^{23} molecules. Find the amount of energy released, in joules, in the fusion of one mole of deuterium molecules.

$(23.8 \text{ MeV/molecule})$
$\times (6.02 \times 10^{23} \text{ molecules/mole})$
$\times (1.6 \times 10^{-19} \text{ J/eV})(10^{6} \text{ eV/MeV})$
$= 2.29 \times 10^{12} \text{ J/mole}$

c. When one mole of deuterium burns, it releases 2.9×10^6 J. How many moles of dueterium molecules would have to burn to release just the energy released by the fusion of one mole of deuterium molecules?

$$\frac{2.29 \times 10^{12}\ \text{J}}{2.9 \times 10^6\ \text{J/mole}} = 7.9 \times 10^5\ \text{moles}$$

23. The energy released in the fission of one atom of $^{235}_{92}U$ is 2.00×10^2 MeV. One mole of uranium atoms, 6.02×10^{23} atoms, has a mass of 0.235 kg.

a. How many atoms are in 1.00 kg $^{235}_{92}U$?

$$\frac{(6.02 \times 10^{23}\ \text{atoms/mole})(1.00\ \text{kg})}{(0.235\ \text{kg/mole})} = 2.56 \times 10^{24}\ \text{atoms.}$$

b. How much energy would be released if all the atoms in 1.00 kg of $^{235}_{92}U$ underwent fission?

$$(2.56 \times 10^{24}\ \text{atoms/kg})(2.0 \times 10^2\ \text{MeV/atom})(1.6 \times 10^{-19}\ \text{J/ev})(10^6\ \text{eV/MeV}) = 8.2 \times 10^{13}\ \text{J}$$

c. A typical large nuclear reactor produces fission energy at a rate of 3600 MW. How many kilograms of $^{235}_{92}U$ are used each second?

$$\frac{3.6 \times 10^9\ \text{J/s}}{8.2 \times 10^{13}\ \text{J/kg}} = 4.4 \times 10^{-5}\ \text{kg/s}$$

d. How much $^{235}_{92}U$ would be used in one year?

$$(4.4 \times 10^{-5}\ \text{kg/s})(3.15 \times 10^7\ \text{s/y}) = 1.4 \times 10^3\ \text{kg/y}$$

Supplementary Problems (Appendix B)

1. The carbon isotope, $^{12}_{6}C$, has a nuclear mass of 12.000 000 u.

a. What is the mass defect of this isotope?

$$\begin{array}{lr}
^{12}_{6}C \quad 6(^1_1p) = 6(1.007\ 825\ \text{u}) = & 6.046\ 950\ \text{u} \\
6(^1_0n) = 6(1.008\ 665\ \text{u}) = & \underline{6.051\ 990}\ \text{u} \\
\text{total mass of nucleus} & 12.098\ 940\ \text{u} \\
^{12}_{6}C & \underline{-\ 12.000\ 000}\ \text{u} \\
\text{mass defect} & -\ 0.098\ 940\ \text{u}
\end{array}$$

b. What is the binding energy of its nucleus?

$$\begin{aligned} E &= (931.5\ \text{MeV/u})(0.098\ 940\ \text{u}) \\ &= 92.16\ \text{MeV} \end{aligned}$$